

Course Title: Complex and Special Functions  
Date: 2011 (2<sup>nd</sup> term)Year: 2<sup>nd</sup> (comp-)  
Allowed time: 3hrsProblem number (1) (32) M

(a) show that  $1 + \cos \theta + \cos 2\theta + \dots + \cos n\theta = \frac{1}{2} + \frac{\sin(n+\frac{1}{2})\theta}{2\sin\frac{\theta}{2}}$

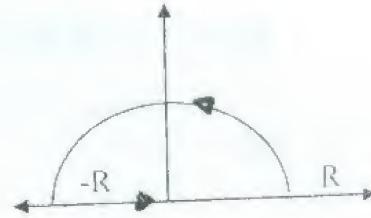
(c) Find all values of  $\sinh(2+2i)$ .(d) If  $f(z) = u + iv$  is analytic and  $u = \text{constant}$  then  $f'(z) = \text{constant}$ .Problem number (2) (32) M

(a) Evaluate  $\int_C \frac{z^3 + z + 6}{(z-1)(z-4)} dz$ ,  $C : |z| = 3$ .

(b) Evaluate  $\int_C \frac{\cosh 3z}{(z-i)^3} dz$ ,  $C : |z-1|=5$ .

(c) Evaluate

$$\int_{-\infty}^{\infty} \frac{\cos 3x}{x^4 + 4} dx \quad \text{On the contour } (R \rightarrow \infty)$$

Problem number (3) (49) M

a) Show that

$$(i) J_0'''(x) = \frac{J_0(x)}{x} + \left(\frac{2}{x^2} - 1\right) J_0'(x)$$

$$(ii) \beta\left(n + \frac{1}{2}, \frac{1}{2}\right) = \frac{(2n)!}{2^{2n}} \frac{\pi}{(n!)^2}$$

(b) Evaluate (i)  $\int_3^{\infty} e^{6x-x^2} dx$

(ii)  $\int_0^2 x \sqrt[4]{(16-x^4)} dx$

(iii)  $\int_0^1 x(1-x^2) J_0(kx) dx$

(c) Find Fourier-Bessel Series of

$$f(x) = \frac{1}{8}(1-x^2) \quad 0 \leq x \leq 1, \quad J_0(\mu_k x) = 0, \quad f(x) = \sum_{k=0}^{\infty} A_k J_0(\mu_k x)$$

Problem(4) (37M)

(a) Define and illustrate with examples:

Fuzzy set – The concentration of a fuzzy set .

(b) Let A be a fuzzy set defined by

$$A = \frac{0.5}{x_1} + \frac{0.4}{x_2} + \frac{0.7}{x_3} + \frac{0.8}{x_4} + \frac{1}{x_5} . \text{ List all } \alpha\text{-cuts and strong } \alpha\text{-cuts of A.}$$

Find the core of A.

(c) By examples illustrate the following statement: In a fuzzy set

Operations, the laws of contradiction and excluded middle are  
not applicable in general .

(d) Given P and Q are two relations

from A to B and from B to C respectively defined by

$$M_P = \begin{bmatrix} 0.3 & 0.5 & 0.8 \\ 0 & 0.7 & 1 \\ 0.4 & 0.6 & 0.5 \end{bmatrix}, \quad M_Q = \begin{bmatrix} 0.9 & 0.5 & 0.7 & 0.7 \\ 0.3 & 0.2 & 0 & 0.9 \\ 1 & 0 & 0.5 & 0.5 \end{bmatrix}$$

calculate  $M_{P \circ Q}$